Low Frequency Leaky Regime in Covered Multilayered Striplines

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Abstract— **This work studies the low frequency leakage of power in covered multilayered striplines including the possibility of several conductors on different interfaces. The appearance and features of the lateral radiation is discussed, assuming that only one parallel-plate waveguide mode is above cutoff. Comparison with quasi-TEM results shows that some leaky waves behave like quasi-TEM modes with radiation losses. Lateral radiation and ohmic conductor losses are also compared.**

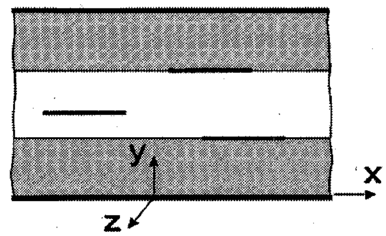

Fig. 1. Cross section of a covered noncoplanar multilayered stripline.

I. Introduction

Owing to its versatility, noncoplanar and/or multilayered striplines have been often proposed in the CAD of microwave integrated circuit (MIC) and monolithic microwave/millimeter wave integrated circuits (MMIC) [1]–[3]. At low frequencies, these lines are usually analyzed using a quasi-TEM approach, which can account for losses due to the use of lossy substrates [4] and/or finite conductivity metallizations [5]. Noncoplanar and/or multilayered structures may also exhibit radiation losses due to the excitation of surface and volume waves in the background waveguide (i.e., the remaining waveguide when all the strips are removed) [6]–[8]. From a technological point of view, the presence of these radiation losses can present serious problems for the proper performance of the systems. On the other hand, the leaky phenomenon could be eventually used for designing some microwave components (antennas, directional couplers, etc.). In any event, a proper study of the propagation characteristics of these structures at low frequency should consider the possible existence of leakage either to prevent it or to take advantage of it. To our knowledge, little attention has been still paid to its low frequency leaky regime, despite of the possible practical significance of the lateral radiation at low frequencies in these types of structures.

In this paper we restrict ourselves to treat the low frequency leakage of power by the dominant waveguide mode in *covered* noncoplanar and/or multilayered striplines. In this case, we will not be concerned with the aspects related to the presence of multiple poles near the real axis and/or branch cuts of the spectral Green's function. The numerical results of the full-wave analysis will be compared with those derived from a quasi-TEM analysis. This comparison suggests that, under certain conditions, the leaky mode can be considered as a laterally radiative mode whose near-field behavior is well described by the quasi-TEM solution. An additional comparison between typical values of leaky and conductor losses will show that the leaky losses can become as important as the ohmic losses (or even more).

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II. Full Wave Analysis

The structure under study is a covered multilayered and/or multiconductor stripline as shown in Fig. 1. An electromagnetic wave with the following dependence: $\mathbf{E}(\mathbf{r},t) = \mathbf{E}(x,y)\exp[-j(k_z z - \omega t)]$, is assumed to propagate in the line; where ω is the angular frequency and k_z the complex propagation constant: $k_z = \beta_z + j\alpha_z$ (with $\beta_z > 0$ and $\alpha_z \leq 0$ being real numbers for a field propagating along the $+z$ direction). The method of analysis employed in this work is an extension of the Galerkin method in the spectral domain modified to take into account the changes in the inversion-contour definition of the spectral Green's dyad [7]. For low frequency leaky waves, when only the dominant waveguide mode is above cutoff, this inversion contour should be deformed to surround the Green's dyad pole associated with this dominant background waveguide mode [6], [7], and [9]. In this case, the leakage of power due to the excitation of the first waveguide mode is radiated at an angle θ from the strip as a nonuniform mode with a complex propagation constant

$$\mathbf{k} = \beta_g \mathbf{u} + j\alpha_g \mathbf{v} \tag{1}$$

where β_g and α_g are real and positive numbers; and $\mathbf{u}$ and $\mathbf{v}$ real and unit vectors in the (x,z) plane. In lossless media, this mode propagates along direction $\mathbf{u}$ and increases at the perpendicular one, $\mathbf{v}$, namely: $\mathbf{u}\cdot\mathbf{v} = 0$ [10]. Therefore $\mathbf{v}$ and $\mathbf{u}$ can be used to form the following right-handed set of unit and orthogonal vectors: $\{\mathbf{v}, \mathbf{a}_y, \mathbf{u}\}$, shown in Fig. 2. The relationships between the line propagation constant and the wavenumber of the excited waveguide mode can be summarized as [10]

$$\beta_z = \beta_g \cos\theta \tag{2}$$

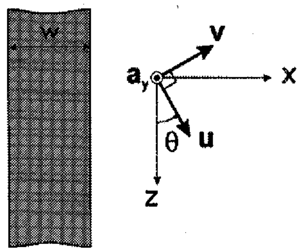

Fig. 2. View of the orthogonal vectors: $\{\mathbf{v}, \mathbf{a}_y, \mathbf{u}\}$.

$$\alpha_z = -\alpha_g \sin\theta \tag{3}$$

$$\beta_g^2 - \alpha_g^2 = \gamma_g^2 \tag{4}$$

where γ_g^2 is the squared wavenumber of the first waveguide mode, which is a real and positive number for lossless media [11].

The real part of the Poynting vector in the transverse (x, z) plane, $\mathbf{S}_t$, of the excited waveguide mode can be computed taking into account the LSM nature of this mode. After a straightforward calculation (see Appendix in [7]), the following expression is obtained

$$\mathrm{Re}(\mathbf{S}_t) = \frac{\gamma_g^2}{2\omega\epsilon\beta_g} |H_v|^2 \, \mathbf{u} \tag{5}$$

where H_v is the magnetic field component in the $\mathbf{v}$ direction; that is, in the direction perpendicular to propagation. In a similar way, the imaginary part of $\mathbf{S}_t$ is found to be

$$\mathrm{Im}(\mathbf{S}_t) = -\frac{\alpha_g}{\beta_g} \frac{\gamma_g^2}{2\omega\epsilon\beta_g} |H_v|^2 \, \mathbf{v} \, . \tag{6}$$

These two equations show that a) there is no flux of reactive power along the direction of propagation, $\mathbf{u}$, of the excited waveguide mode; and b) there always exists a flux of active power in this direction, provided that $\gamma_g^2 > 0$.

III. Quasi-TEM Analysis

A quasi-TEM analysis has also been used in order to compare with the full-wave results. This analysis assumes the usual quasi-TEM approach, that is, the longitudinal component of both the electric and magnetic fields, E_z and B_z, have been neglected, and the transverse fields have been expressed as the transverse gradients of some potential functions (the propagation constant can be then expressed as a function of line capacitance and inductance per unit length). As is well known, this analysis can not evaluate the radiation losses due to the leakage, but after some modifications (see for instance [4] and [5]), the ohmic losses in both the substrate and conductors can be evaluated with high accuracy in the low frequency range (i.e., when the line wavelength is much greater than the transverse dimension of the line). The details of the quasi-TEM analysis can be found in [5], and will not be presented here.

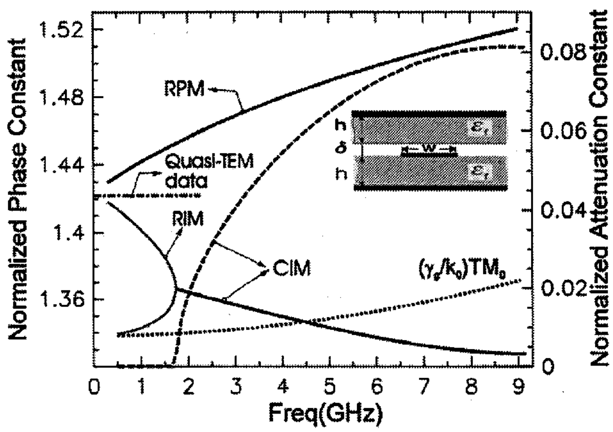

Fig. 3. Normalized propagation constants of a stripline with ans airgap versus frequency. $\delta = 3.5$ mm, $h = 4.45$ mm, $w = 6.35$ mm, $\epsilon_r = 2.6$. Solid lines: Normalized phase constants of proper and improper stripline modes. Dashed line: Normalized attenuation constant. Dotted line: Normalized wavenumber of the dominant parallel-plate mode.

IV. Numerical Results

Our numerical computations have been checked by comparing with pertinent previously published results. We found a good agreement with the data reported in Fig. 2 of [8]. This result was shown in [7] and now this structure is analyzed versus frequency in Figs. 3 and 4. At low frequencies and for the chosen dimensions, Fig. 3 shows that the normalized (to $k_0 = \omega\sqrt{\epsilon_0\mu_0}$) phase constant of the quasi-TEM solution is higher than the normalized eigenvalue, γ_g/k_0, of the first (and only above-cutoff) waveguide mode and very close to the phase constant of a bound real proper mode (RPM). A complex improper mode (CIM) (namely, a leaky mode) is also present for frequencies above 1.6 GHz after the conjunction of two real improper modes (RIM). When frequency increases, the quasi-TEM data appear approximately at the same distance from the phase constants of the CIM and the RPM. This points out that the quasi-TEM solution does not account properly for any of the full-wave solutions in the present case (as was expected in view of the h/λ ratio of this structure). It is interesting to note in Fig. 3 that the onset of the leaky mode occurs with a phase constant greater than the wavenumber of the first waveguide mode γ_g, meaning that the solution lies in the region that is usually called the *spectral gap* [12]. Fig. 4 shows the normalized phase constant, β_g/k_0, and the radiation angle, θ, of the excited waveguide mode in addition to the normalized phase constant, (β_z/k_0), of the stripline leaky mode. It can be seen that the leaky wave is fast with respect to that of the excited waveguide mode for all frequencies (regarding the phase velocities: $\beta_z < \beta_g$), even in the spectral gap region ($\beta_z > \gamma_g$), and that the radiation angle takes nonzero values from the onset of the leaky mode (where the leakage begins). The leaky wave does not show any particularly peculiar behavior from its onset, even inside the spectral gap region, and therefore the question of the excitability of such a leaky wave by a finite source (which relates to the *physical meaning* of the wave) can not be answered in the frame of a purely (2D) propagative analysis—as performed in this paper. However, considering the analysis of a simpler structure [13], it can be

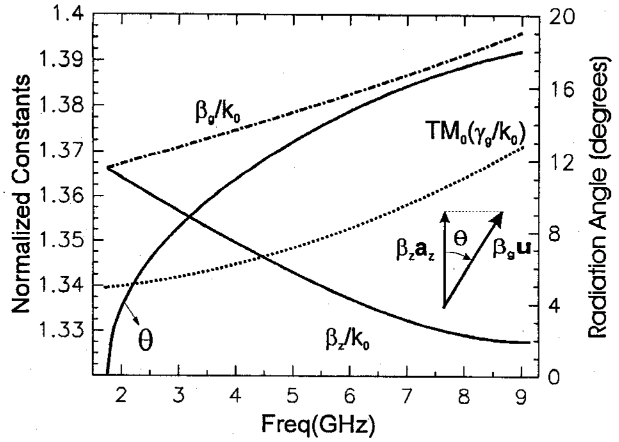

Fig. 4. Dispersion curves and radiation angle for the leaky mode of Fig. 3.

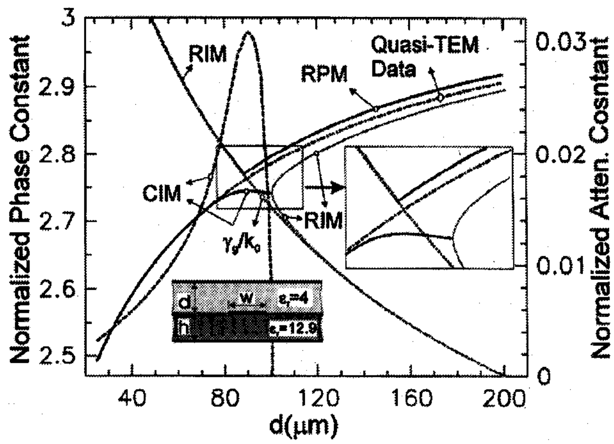

Fig. 5. Normalized propagation constants of an inhomogeneous stripline versus the upper ground plane height at 2 GHz. $h = 200$ μm, $w = 80$ μm. Solid lines: normalized phase constants of proper and improper stripline modes. Dashed line: Normalized attenuation constant. Dotted line: Normalized wavenumber of the dominant parallel-plate mode (γ_g/k_0). Dashed-dotted line: Quasi-TEM data.

postulated that the degree of excitation of the leaky wave on the line by a finite source decreases as the solution enters into the spectral gap region [13].

Next, we are interested in analyzing what happens when the quasi-TEM solution for the phase constant becomes smaller than the wavenumber of the first waveguide mode γ_g. For this purpose we have chosen the dimensions of the inhomogeneous stripline and the frequency as shown in Fig. 5. A small ratio between the wavelength and the transverse dimensions of the structure, $(d+h)/\lambda_0 \sim 0.01$, has been also imposed to assure both the validity of the quasi-TEM approach and that only one waveguide mode is above cutoff. Also note that this structure can be used to analyze one of the two fundamental modes (the even mode) of a noncoplanar symmetrical broadside coupled line. In the inhomogeneous stripline, a leaky mode can be induced varying the height of the upper ground plane. This leaky mode turns into two RIM's for $d > 101$ μm and the bound mode (RPM) becomes a RIM after its phase constant reaches the wavenumber of the dominant waveguide mode ($d \sim 88$ μm). A comparison with the quasi-TEM data clearly shows that the phase constant of the leaky mode coincides with the quasi-TEM results at the lower values of the upper ground plane height. In this range $\alpha_z \ll \beta_z$, and therefore from (2) to (4): $\alpha_g \ll \beta_g$ (these facts have been also tested numerically), implying that the first waveguide mode is excited as a quasiuniform mode. In this case (5) and (6) show that the flux of reactive power—at any direction—is negligible in comparison with the flux of active power, and consequently the nature of this low frequency leaky mode should be considered radiative rather than reactive. The closeness of the different full-wave solutions to the quasi-TEM solution in the different d ranges suggest that, for $d > 100$ μm the real proper mode has a field pattern very similar to the quasi-TEM solution, and for $d < 80$ μm it is the leaky wave field that resembles the quasi-TEM field pattern.

An interesting feature observed in Fig. 5 is that the quasi-TEM results appear closer to the phase constant of the leaky mode than to the phase constant of the bound mode in its respective ranges. This fact can be explained taking into account the small values of the normalized attenuation constant of the leaky mode in Fig. 5 for all values of d, since these small values suggest that the first waveguide mode is excited with a small amplitude. In other words, the contribution of this waveguide mode to the near field is small and we can assume that the near field of the leaky mode is mainly a superposition of all the remaining nonuniform waveguide modes, which are well below cutoff at the assumed low frequencies. Therefore the eigenvalues of these latter modes are purely imaginary numbers with absolute values much higher than the leaky mode propagation constant, $|\gamma_{g,n}| \gg |k_z|, \omega\sqrt{\mu\epsilon}$ $(n \neq 0)$. Under these circumstances, the differential operator of the Helmholtz equation satisfied by these waveguide modes, $\partial^2/\partial y^2 - k_x^2 - k_z^2 + w^2\mu\epsilon$, approaches $\partial^2/\partial y^2 - k_x^2$, since $w^2\mu\epsilon \ll k_x^2$ and $k_z^2 \ll k_x^2$. The Helmholtz operator can be then approximated by the Laplace's operator, $\partial^2/\partial y^2 - k_x^2$, for all the meaningful near-field components of the leaky wave. On the other hand, when the quasi-TEM solution accounts for the bound mode of Fig. 5, the contribution of the first waveguide mode to the expansion of the near line field does not seem to be negligible, making the line field less "TEM" than in the leaky case.

A result showing the appearance of leaky modes in broadside-coupled structures is presented in Fig. 6. This figure shows the behavior of the normalized propagation constants versus the value of the relative permittivity of the lowest substrate. Although this structure has two quasi-TEM modes, we have not plotted that mode corresponding to the highest value of the phase constant (even-like mode) because here this mode shows the common behavior of a bound mode. However, the other quasi-TEM solution (QTS) appears very close to either a leaky mode or a real proper mode. The phase constant of the RPM runs very close to the dispersion curve for the first waveguide mode for higher values of ϵ_r, and approaches the QTS at the low values of the relative permittivity. In the first region, the RPM seems to be a perturbation of the waveguide mode caused by the presence of the strips. We can deduce from the curves of Fig. 6 that the appearance of the leaky mode is again related to the closeness of the quasi-TEM solution and

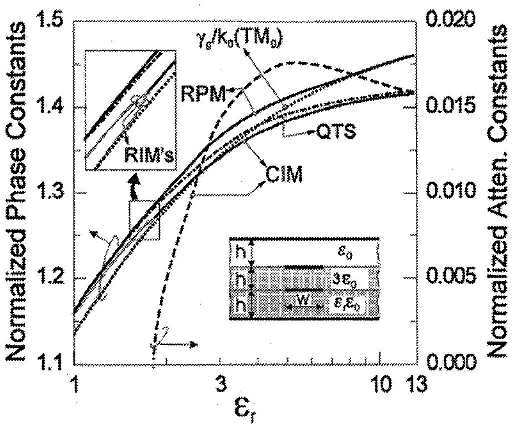

Fig. 6. Normalized propagation constants of an broadside-coupled line as a function of the relative permittivity of the lower substrate. $h = w = 200$ μm, Freq $= 2$ GHz. Solid lines: Normalized phase constants of proper and improper modes. Dashed line: Normalized attenuation constant; Dotted line: normalized wavenumber of the dominant parallel-plate mode (γ_g/k_0). Dashed-dotted line: Quasi-TEM data.

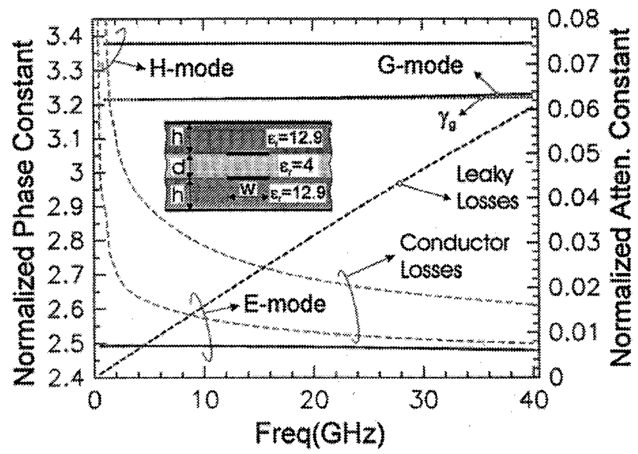

Fig. 7. Dispersion curves of the fundamental modes of a broadside-coupled microstrip line, $h = 200$ μm, $w = 80$ μm, $d = 50$ μm. Solid lines: normalized phase constants of the three fundamental full-wave modes. Dashed lines: normalized attenuation constant. Dotted line: normalized wavenumber of the dominant parallel-plate mode (γ_g/k_0).

the wavenumber of the dominant waveguide mode, although the onset of the leaky regime cannot be easily predicted. At the highest permittivity values (with the QTS being lower than the waveguide wavenumber), we find that the full-wave solution that is closer to the QTS is the leaky mode. Since the attenuation constant of this mode is much lower than the phase constant, the leaky mode can be considered again as a laterally radiative mode whose near-field behavior resembles a quasi-TEM mode.

Fig. 7 shows the propagation constants of a potentially leaky symmetrical broadside-coupled line as a function of frequency. The suitability of a quasi-TEM approach could be again expected in view of the reduced dimensions of this structure; this is also confirmed by the nondispersive nature of the full-wave solutions for the phase constant plots of both the odd mode (E-mode) and even mode (H-mode). In fact, a quasi-TEM analysis of this structure (assuming perfect conductors) yields almost identical results for the phase constants. Nevertheless, for small distances between strips, it has been found that the E-mode of this structure becomes leaky starting at very low frequencies, as can be seen in Fig. 7. The attenuation constant of this leaky mode shows an almost quadratic dependence with the frequency (linear dependence for the normalized attenuation constant, α_z/k_0). This dependence can be understood taking into account that the attenuation constant must be an even function of the frequency [14]. This latter fact together with the linear dependence of the phase constant with frequency suggests the validity of a first-order approach in the series expansion of the fields in powers of frequency. Therefore, for practical purposes, only one full-wave computation would be required to obtain both the phase and attenuation constants of a leaky mode for all frequencies where the quasi-TEM approach yields very accurate results for the phase constant. This accuracy seems to be assured for small dimensions of the line when the quasi-TEM solution is well below than the wavenumber of the first waveguide mode.

An additional bound mode, referred as G-mode in the figure, also appears very close to the dispersion curve of the first waveguide mode (this G-mode seems to be a perturbation of the waveguide mode). Fig. 7 also provides a comparative analysis between the leakage losses of the E-mode and the conductor losses induced by the presence of real copper strips ($\sigma = 4 \cdot 10^7 (\Omega\text{m})^{-1}$) with finite thickness ($t = 5$ μm). The results for conductor losses have been computed using the method developed in [5]. This comparison shows how ohmic losses, which are dominant at very low frequencies, can be lower than lateral radiation losses, even at frequencies where the phase constant of the leaky wave is still very close to the quasi-TEM solution.

V. Conclusion

Low frequency leakage of power in covered noncoplanar and/or multilayered stripline transmission lines has been analyzed. It has been verified that the low frequency leaky regime is expected to appear when the quasi-TEM values for modal phase constants appears near (or below) the wavenumber of the dominant waveguide mode. In this circumstance, the computed results for the phase constant of the leaky mode have been compared with the quasi-TEM results, showing a good agreement over a wide frequency range. This behavior has been systematically found for small dimensions of the structure when the quasi-TEM phase constant is smaller than the wavenumber of the first waveguide mode. These results show that the low frequency leaky modes behave in these structures like a quasi-TEM mode with radiation losses.

Computed values for the power associated with the low frequency leaky mode have also revealed that its nature is mainly radiative instead of reactive. A comparative analysis between radiation and conductor losses has also shown that, in many cases, radiation losses can be higher than conductor losses. Therefore, the characterization of the losses of the structure should include a full-wave analysis accounting for the possible leakage of power. Nevertheless, when the phase constant is well approximated by the quasi-TEM results, we have found a quadratic dependence of the attenuation constant with frequency (this fact is somewhat expected after a series expansion of the field in powers of frequency), which implies that only one full-wave computation would provide the propagation constants over a wide frequency range.

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